Distributed Generation as Voltage Support for Single Wire Earth Return Systems

M. A. Kashem, *Member, IEEE,* and Gerard Ledwich, *Senior Member, IEEE*

***Abstract*—Key issues for distributed generation (DG) inclusion in a distribution system include operation, control, protection, harmonics, and transients. This paper analyzes two of the main issues: operation and control for DG installation. Inclusion of DG in distribution networks has the potential to adversely affect the control of voltage. Both DG and tap changers aim to improve voltage profile of the network, and hence they can interact causing unstable operation or increased losses. Simulations show that a fast responding DG with appropriate voltage references is capable of reduction of such problems in the network. A DG control model is developed based on voltage sensitivity of lines and evaluated on a single wire earth return (SWER) system. An investigation of voltage interaction between DG controllers is conducted and interaction-index is developed to predict the degree of interaction. From the simulation it is found that the best power factor for DG injection to achieve voltage correction becomes higher for high resistance lines. A drastic reduction in power losses can be achieved in SWER systems if DG is installed. Multiple DG can aid voltage profile of feeder and should provide higher reliability. Setting the voltage references of separate DGs can provide a graduated response to voltage correction.**

***Index Terms*—Dynamics, eigenvalues and eigenfunction, power distribution, power generation control, sensitivity, voltage control.**

I. Introduction

THE demand for distributed generation (DG) implementation has potential for significant growth in areas where combined heat and power can be used, in locations with poor quality of supply, or where the losses or cost of reinforcement are high. DG installation benefits utilities by releasing transmission and distribution capacity, deferring of new or upgraded T&D infrastructure, improving system reliability, and reducing power losses, as well as assisting customers by satisfying their instantaneous power demands, supporting customer voltage, and improving power quality. Different components of these benefits are drivers for a growth in distributed generation worldwide.

Distribution systems are generally designed to operate without any generation on the distribution system or within customer premises. The introduction of DG sources on the distribution network can significantly impact the power-flow and voltage conditions for customers and on utility equipment. This impact can be positive or negative depending on the operating conditions of the distribution system and DG [1]. The main technical issues arising from the connection of DG to distribution networks include protection, dynamic interaction/voltage control, transient/small signal stability, quality of supply, and harmonics issues. The authors of [2] have studied the impacts of DG upon transmission system transient and small-signal stability. This paper addresses the dynamic interaction and voltage control issues through extensive simulation of single wire earth return (SWER) systems. A SWER system is, by definition, a single-wire distribution system in which all equipment is grounded to earth and the load current returns through earth. It is typically constructed in rural areas as an expansion of a three-phase system. Lines of this system are typically long, often resulting in the current having a leading power factor at light loads. Loads are light and load density is typically 0.5 kVA/km with an average maximum demand per customer of 3.5 kVA in a SWER system [3]. An isolation transformer is used to connect the SWER lines with the three-phase supply and to isolate earth currents of the SWER system from the three-phase mains supply. Losses of SWER systems are high due to the high resistance of the SWER conductors. SWER voltages used in Queensland, Australia, are typically 12.7 and 19.1 kV.

The connection of DG has a very real potential of creating dynamic interaction with transformer tap changer in distribution systems. The number or frequency of tap changer events per day is usually minimized in order to prolong the life of the tap changer [4]. The traditional analysis techniques have not addressed the dynamic interaction created by tap changer and DG, or DG with another DG in sufficient detail, and no extensive simulation has been conducted on this interaction. Currently, the only small-signal analysis tool widely available is time-domain eigenvalue analysis, which provides only partial information on the power system dynamics [5]. However, the authors in [6] have assessed the interaction and small-signal stability of DG controllers by using eigenvalue analysis and verified the results using individual channel analysis and design (ICAD). The authors of [7] have presented an optimal load shedding strategy for power systems with multiple DGs. In [8] some conflicts with the operation of distribution systems due to DG installation have been discussed. The authors of [9] have proposed a new voltage regulation coordination method of DG system to coordinate DG with tap changer and line drop compensator.

In this paper, effective operation and control of DG based on a synchronous generator are discussed and voltage sensitivity analysis is performed. Dynamic interaction between tap changers and DG is investigated and a control is proposed to alleviate any interaction. The interaction between DG controllers is examined using eigenanalysis, and a voltage-interaction index

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M. A. Kashem is with the School of Engineering, University of Tasmania, Hobart, Australia.

G. Ledwich is with the School of Electrical and Electronic Systems Engineering, Queensland University of Technology, Brisbane, Australia.

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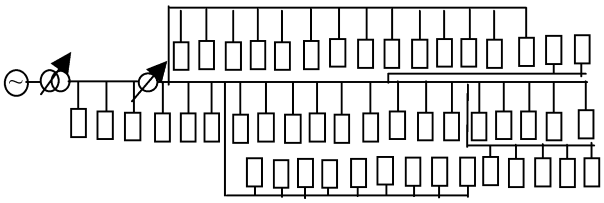

Fig. 1. SWER model.

is derived to determine the potential for problems. Also, a DG controller is designed based on voltage sensitivity to operate in real (P)-reactive (Q) generation with Q priority and improve the network voltage efficiently.

II. Modelling of SWER Network

A SWER system usually comprises SWER isolator, regulator, or tap changer, automatic circuit recloser (ACR), single-phase feeder, subfeeder and loads. Fig. 1 shows the structure of the section of an idealized network relevant to this paper. The mains supply at the connection of SWER recloser is modeled by its Thevenin equivalent. The Thevenin voltage is assumed the same as the substation voltage and Thevenin impedance is obtained from short-circuit megavoltampere (MVA) level at that point. Uniform load distribution is assumed for the entire feeder. A typical SWER system could be based on a voltage regulator with $\pm 10\%$, 32-5/8% steps, and (OLTC) transformer with $\pm 5\%$, 4-2.5% steps to boost or buck the voltage level. However, in some SWER systems OLTC transformers are not cost effective and a fixed step is used. If it is required to change the step position due to the heavy loading, somebody needs to climb up the tower and change the tap position manually. As the study is to determine the interaction between tap operation and DG, to see the effect in worst case situation of tap operation the automatic operation of tap position for both, voltage regulator and OLTC transformer is considered during simulation. In the fixed tap situation, there will be less risk for interaction compared to the case studied in this paper.

III. Voltage Sensitivity Based Cost-Effective DG Operation and Control

Control of P and Q for a single DG operation can be determined from the following equation, where the performance index J includes a tradeoff between real generation and voltage errors

$$J = k_1 \Delta V^2 + k_2 P. \tag{1a}$$

Equation (1a) can be expanded as

$$\Delta J = k_1 \Delta V \frac{\partial V}{\partial Q} \Delta Q + k_1 \Delta V \frac{\partial V}{\partial P} \Delta P + k_2 \Delta P. \tag{1b}$$

Taking this sensitivity around $P = 0$, $Q = 0$ and assuming the DG injection does not create nonlinear effects, $\Delta P = P$, $\Delta Q = Q$

$$J \approx k_1 \Delta V \frac{\partial V}{\partial Q} Q + k_1 \Delta V \frac{\partial V}{\partial P} P + k_2 P. \tag{1c}$$

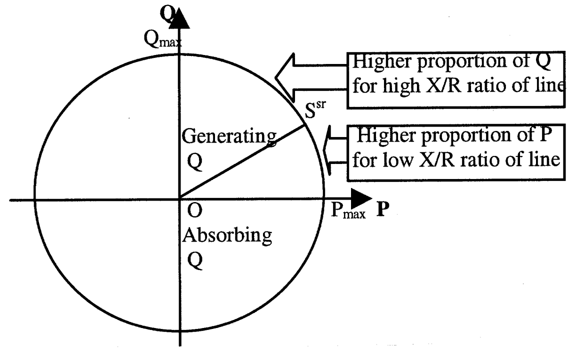

Fig. 2. Effective region of DG operation.

Rearranging the above equation, we obtain

$$J = k_1 \left[\Delta V \frac{\partial V}{\partial Q} Q + \left(\Delta V \frac{\partial V}{\partial P} + \frac{k_2}{k_1} \right) P \right] \tag{1d}$$

where $P^2 + Q^2 = S^2$, k_1 = dollar penalty associated with voltage change, k_2 = fuel cost factor associated with real power generation, S = size of DG (kVA), and J = cost. The term $k_2 P$ in (1a) represents the actual generation cost, whereas the term $k_1 \Delta V^2$ is a more indirect measure of the undesirability of large voltage deviation. In practice, the value of k_1 would be adjusted until the appropriate balance between voltage deviation and generation cost is achieved. Here P is assumed positive while $\Delta V \partial V / \partial Q$ and $\Delta V \partial V / \partial P$ are typically negative. This problem is of the form $J = \alpha Q + \beta P$, which has the optimal solution

$$P = \frac{S}{\sqrt{1 + \left(\frac{\alpha}{\beta}\right)^2}}$$

$$Q = \frac{S}{\sqrt{1 + \left(\frac{\beta}{\alpha}\right)^2}}$$

where

$$\frac{\alpha}{\beta} = \frac{\left(\frac{\partial V}{\partial Q}\right)}{\left(\frac{\partial V}{\partial P} + \frac{k_2}{\Delta V\, k_1}\right)}$$

under the assumption that β is negative. As the cost of fuel rises, the negative term $\Delta V \partial V / \partial P$ becomes partly cancelled by k_2/k_1 and α/β becomes larger, and thus the real power P to optimally correct the voltage becomes small. Similarly, as the voltage sensitivity to Q decreases, the optimal solution biases to a pure real power correction. In this situation, the cost J will be optimized if the real power generation component of the injection is increased. As ΔV increases the optimal solution moves toward the low fuel cost solution. A study of different power lines shows that for high X/R lines, reactive power injection is most effective to improve voltage profile, whereas for low X/R, real power injection helps appreciably to support voltage and improve voltage profile (where R and X are line resistance and reactance, respectively) [10]. Fig. 2 indicates the DG operating regions in which DG may generate reactive power for heavy loading situations or absorb reactive power for a light load condition. For maximum voltage improvement during heavy loading, the DG operating point should be in the upper half region of first quadrant, as a high X/R ratio of line is optimized for a higher proportion of Q generation and it would be in the lower half for a low X/R ratio. As discussed above, the values for $\partial V/\partial P$ are seen to be much higher than $\partial V/\partial Q$ for low X/R lines and thus for low values of fuel cost will tend to give high P solutions. If the fuel cost is low then the best P: Q point is in the ratio of $\partial V/\partial P$: $\partial V/\partial Q$. When fuel cost is high the optimal solution is for pure Q correction wherever possible, moving to the S^{sr} ratio for large ΔV.

A. Determination of Voltage Sensitivity

Voltage sensitivity of distribution lines plays a predominant role for real and reactive power injection and DG operation. The ratios of sensitivity are different for different lines due to the resistance and reactance of the lines. If real and reactive powers are injected at the ratio of optimal voltage sensitivity of lines, the maximum voltage improvement can be achieved. Therefore, a sensitivity study should be performed to determine the ratio of optimal sensitivity before operating the DG. One of the options to perform the computation of voltage sensitivity is incremental power injection. The procedure for this method is given below.

i) Inject real power only (X kW) into the network at a peak time load (as DG will be operated for peak shaving).
ii) Compute the change in voltage at the point of injection.
iii) Inject reactive power (X kVAr) into the network for the same load.
iv) Compute again the change in voltage.
v) Calculate the ratio of voltage changes due to the above real and reactive injections, which is the ratio of voltage sensitivity.

IV. DESIGN OF DG CONTROLLER

The DG controller has been designed from the concept that changing the fuel injection will vary the real power while changes in field excitation will vary the reactive power from the DG. The control design is basically a P-I controller while the generator/motor response is modeled as a low-pass filter. Equations (2) and (3) show how the voltage error is controlled by P-I controller. For the case of DG-P (DG using only real injection) controller, field excitation is kept nearly constant and input power controlled to produce the required DG real power; whereas, in the case of DG-Q (DG using only reactive injection) controller it is only the field control which is used

$$\Delta V = V_{act} - V_{ref} \tag{2}$$

$$U = k_p \Delta V + k_i \int \Delta V \tag{3}$$

where V_{act} and V_{ref} are the actual voltage at the DG connection and the prespecified reference voltage to be achieved by the DG, respectively. k_p and k_i are the proportional and integral constants, respectively. ΔV is the error voltage and U is the controller generated signal.

The ratio of voltage sensitivity to real and reactive injection for a given system can be represented as in (4). The sensitivity ratio is used in the design of DG-PQ (DG with real and reactive injection) controller to guide the P and Q generation to inject real and reactive powers for maximum voltage improvement

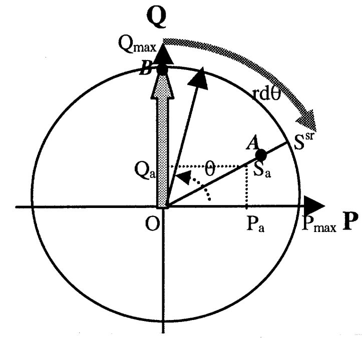

Fig. 3. DG operation and power injection based on voltage sensitivity.

$$S^{sr} = \frac{\frac{\partial V}{\partial P}}{\frac{\partial V}{\partial Q}} = \frac{\beta}{\alpha}\bigg|_{k_2=0}. \tag{4}$$

As k_2 is the fuel cost factor associated with real power generation, k_2 approaching zero or high ΔV means that fuel cost is low compared with the penalty for voltage excursions. Therefore, (4) refers the situation where the fuel cost effect is low or voltage error effect is high.

At the steady-state condition, the optimal solution is $P = \gamma Q$, where γ is the maximum sensitivity, and in the special case of $k_2 = 0$ or high ΔV

$$P = S^{sr} Q. \tag{5}$$

The maximum value of P generation at a given sensitivity is

$$P^{sr}_{\max} = \frac{S^{sr}}{\sqrt{1 + (S^{sr})^2}} S \tag{6}$$

where S is the size of DG (kVA) and S^{sr} is the ratio of sensitivity of voltage to real power and reactive power of DG. This is the optimal use of a given generator for low fuel cost or large voltage error.

A. Transition of DG Operation From Q to PQ Mode

For almost all technologies except wind and solar the incremental cost of Q generation is lower than that of P generation. Inverter interfaced DGs (e.g., microturbine, PV) may be able to inject pure Q or pure P. Fuel cost is high for some DG technologies, and therefore, more emphasis is given to injection of Q rather than P to improve voltage profile. However, some technologies may not offer pure Q generation. In those situations, DG controllers will be activated in such a way that DGs will generate the maximum proportion of Q with minimal value of P. A DG-PQ needs to be operated on the line OS^{sr} to maintain the same sensitivity as shown in Fig. 3. It is noted that DG-PQ can be operated at DG-Q generation mode on line OQ to generate pure Q for a limited condition and achieved the same voltage improvement as DG-PQ at low voltage correction. The incremental voltage for a DG operating point (P, Q) can be defined as

$$\Delta V = \beta P + \alpha Q = \alpha \left(S^{sr} P + Q\right) \tag{7}$$

where $\alpha = \partial V/\partial Q$ $\beta = \partial V/\partial P$ and $S^{sr} = \beta/\alpha$.

If A and B are the two operating points of the DG representing the same level of voltage sensitivity, the following condition will be valid:

$$S^{sr}P_a + Q_a = S^{sr}P_b + Q_b. \tag{8}$$

If the injected powers of DG at points A and B represent (P_a, Q_a) and $(0, Q_{\max})$, respectively (shown in Fig. 3), (8) will become

$$S^{sr}P_a + Q_a = Q_{\max}. \tag{9}$$

Using optimum (P, Q) ratio at which $P = \gamma Q$, (9) will give the following condition:

$$P_a = \frac{\gamma Q_{\max}}{1 + S^{sr}\gamma} \quad Q_a = \frac{Q_{\max}}{1 + S^{sr}\gamma}. \tag{10}$$

The above condition is true if points A and B represent the same sensitivity level that gives the same voltage improvement. Therefore, an operating point A (P_a, Q_a) for DG-PQ represents the point B $(0, Q_{\max})$ for DG-Q. All points from O to A on line OS^{sr} can be projected on line OQ. By operating DG-PQ in the DG-Q mode, one can save the operating fuel cost of the DG. This Q-only solution is always optimal until ΔV becomes large. The DG controller generates Q up to the kVA limit and then turns into PQ generation mode by adjusting P and Q generation to improve voltage profile further if necessary. Therefore, it is recommended that for low-level correction a DG operates to generate Q only in the range of Q to $Q_{\max}$ and after $Q_{\max}$ change the DG operating angle slowly toward the line of maximum sensitivity γ, maintaining the constant kVA limit. For this angle change, a nonlinear relationship exists between the voltage and power generation. A gain factor has been introduced below to take correct the nonlinearity and is used in the controller for improved stability.

Let the voltage response to reactive power be X. For this paper we take the real power sensitivity of voltage to be S^{sr} times that of reactive, where S^{sr} is the voltage sensitivity ratio. The voltage sensitivity during this angle change is

$$\begin{aligned} dV &= dq \times X + dp \times S^{sr} \times X \\ &= -rd\theta \cos\theta \times X + rd\theta \sin\theta \times S^{sr} \times X \\ &= rd\theta \left(-\cos\theta + S^{sr} \times \sin\theta\right) X \end{aligned} \tag{11}$$

where dq and dp are the incremental changes of reactive and real generations. In the above equation, $d\theta$ and dq are negative. Rearranging (11) we get

$$\frac{dV}{d\theta} = r\left(-X\cos\theta + S^{sr} \times X \sin\theta\right). \tag{12}$$

This gives a nonlinear relationship between voltage and angle around the circle. Ideally the correction of the nonlinearity can be given by

$$\theta = \theta_0 + K\left(\frac{d\theta}{dV}\right)(V_{DG} - V_{REF}) \tag{13}$$

where K is a gain factor and $r\theta_0$ is the initial peripheral length. However, this gain becomes infinite at the point $(S^{sr}P, Q)$ which gives undesirable characteristics to the closed loop performance. A control-law implemented as

$$\theta = \theta_0 + \frac{K}{r}\left(\frac{1}{-X\cos\theta + \alpha X \sin\theta}\right)(V_{DG} - V_{REF}) \tag{14}$$

where α is chosen as less than S^{sr} performs near-perfect correction but avoids the use of infinite gain.

The decision to start the DG is based on whether the DG connection point voltage is below a threshold for a defined period.

It is noted that fuel efficiency and maintenance issues become a problem especially in operating a diesel generator for long periods below 30% rated power in a practical situation. This is the reason the manufacturers of diesel engines usually stipulate a minimum loading around 25–30% for effective operation. It is possible in the above development to model the controller for diesel engine to operate from 30% real generation. The diesel generator will start voltage correction with 30% of real and zero reactive generation and will increase its reactive generation from zero to the maximum value. For high level of voltage correction, the DG will be operating with maximum kVA and will move to maximum voltage sensitivity level if the voltage reduces further.

V. Control Logic for Controlling the Tap Changer and DG

Inclusion of a distributed generator in distribution networks has the potential to improve the voltage response of the line but there is a risk of an adverse effect on the control of the tap changer. Both the DG and tap changer aim to improve the voltage profile of the network, and hence they can interact causing voltage oscillations or circulating current. The potential for interactions would be stronger if all were continuously acting. It is experimentally seen that delaying the control action of one of them alleviates the dynamic interaction among them. The following control logic can be utilized to control tap changer and DG. By using this, the interaction between them could be minimized.

i) A tap changer requires a delay characteristic to avoid excessive wear of the contacts. In contrast, the DG responds at every instant to control the voltage in the network.
ii) Connection point voltage is used in the feedback controllers for both tap and DG systems.
iii) A 1.5% voltage error dead-band is used for tap changer. The proportional and integral type of controller is used for the DG. The reference level for the DG is kept low to allow the tap changer to supply most of the voltage control in steady state.

VI. Interaction Between DG Controllers

If multiple DGs are installed in close proximity, they may work in opposition to control the local network voltage leading to oscillations or excess circulating current. If they are installed at a considerable distance from each other, their local network voltages do have not much impact on each other due to the line voltage-drop and consumer loads between them. Interaction between distributed generators may be a concern for voltage correction by multiple DGs. Also a DG may create oscillatory waveforms due to low inertia. Proper excitation damper design may be able to stabilize DG in this situation. The analysis for short-term angle stability interaction can be performed following the techniques discussed in [6]. This paper investigates longer term dynamic interaction of voltage controllers assuming the angle has reached its steady state. The following analysis for voltage interaction of DGs has been performed in discrete time domain assuming angle transients have settled and therefore the analysis only gives an indication of only one form of interaction.

A. Derivation of Voltage Interaction-Index

Assume a single line system consists of N number of physical load buses and two distributed generators are connected at locations $N-2$ and $N-6$ with their internal buses labeled $N+1$ and $N+2$, respectively. It is noted that to show the voltage profile against the feeder length, an internal bus for each DG is not shown when an investigation of voltage profile is made. The internal buses of the DGs are labeled at the end to avoid renumbering actual system buses. DG connection points at load buses have been referred as DG buses. Bus voltage and current of this system are related as shown in (15) at the bottom of the page.The matrix (15) has been partitioned into submatrices as shown in (16)

$$\begin{bmatrix} Y_1 & Y_2 & Y_4 \\ Y_2^T & Y_3 & Y_5 \\ Y_4^T & Y_5^T & Y_6 \end{bmatrix} \begin{bmatrix} V_S \\ V_X \\ V_{DG} \end{bmatrix} = \begin{bmatrix} I_S \\ 0 \\ I_{DG} \end{bmatrix} \tag{16}$$

where $Y_1 = [y_{1,1}]$ $Y_2 = [y_{1,2} \quad y_{1,3} \quad \cdots \quad y_{1,N}]$ $Y_4 = [y_{1,N+1} \quad y_{1,N+2}]$

$$Y_3 = \begin{bmatrix} y_{2,2} & y_{2,3} & \cdots & y_{2,N} \\ \vdots & & & \\ y_{N,2} & y_{N,3} & \cdots & y_{N,N} \end{bmatrix}$$

$$Y_5 = \begin{bmatrix} y_{2,N+1} & y_{2,N+2} \\ \vdots & \\ y_{N,N+1} & y_{N,N+2} \end{bmatrix}$$

$$Y_6 = \begin{bmatrix} y_{N+1,N+1} & y_{N+1,N+2} \\ y_{N+2,N+1} & y_{N+2,N+2} \end{bmatrix}$$

$$V_X^T = [V_2 \quad \ldots \quad V_N] \quad V_{DG} = \begin{bmatrix} V_{DG1} \\ V_{DG2} \end{bmatrix} I_{DG} = \begin{bmatrix} I_{DG1} \\ I_{DG2} \end{bmatrix}.$$

From (16) we obtain

$$V_X = -Y_3^{-1}\left(Y_2^T V_S + Y_5 V_{DG}\right). \tag{17}$$

By applying the principle of superposition, the relative changes of voltages due to DG injection can be obtained. By substituting $V_S = 0$ in (17) and examining the response of the system to 1 p.u. voltage at DG1 and DG2 individually, we get

$$V'_{X_{DG1}} = -Y_3^{-1} Y_5 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } V'_{X_{DG2}} = -Y_3^{-1} Y_5 \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{18}$$

The relative changes of voltage magnitudes at DG connection points can be extracted from the matrix in (18) as

$$\mathbf{A}_{DG} = \begin{bmatrix} V^{\text{Relative}}_{DG1_{N-2-1}} & V^{\text{Relative}}_{DG1_{N-6-1}} \\ V^{\text{Relative}}_{DG2_{N-2-1}} & V^{\text{Relative}}_{DG2_{N-6-1}} \end{bmatrix} \tag{19}$$

where $N-2-1$ and $N-6-1$ are the locations of relative changes of voltages for DG1 and DG2 in the matrix of (18). Therefore, the measured voltages at DG connection points in this example system are

$$\mathbf{V}_m = \mathbf{A}_{DG} \mathbf{V}_{DG} \tag{20}$$

where $\mathbf{v}_m = \begin{bmatrix} V_{m1} \\ V_{m2} \end{bmatrix}$, and V_{m1} and V_{m2} are the measured voltages for DG1 and DG2, respectively. For a closed-loop control system with proportional control gain K (where $\mathbf{K} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$, with gains K_1 and K_2 associated with DG1 and DG2, respectively), DG voltages can be obtained as

$$V_{DG1} = -K_1 \left(V_{m1} - V_{ref}\right)$$

and

$$V_{DG2} = -K_2 \left(V_{m2} - V_{ref}\right) \tag{21}$$

where V_{ref} is the reference voltage. DG voltages at predicted future states are

$$\mathbf{V}_{DG_{k+1}} = \mathbf{V}_{DG_k} - \mathbf{K}\mathbf{V}_m - \mathbf{K}V_{ref}. \tag{22}$$

If V_{ref} is assumed zero, (21) can be rearranged as

$$\mathbf{V}_{DG_{k+1}} = (\mathbf{I} - \mathbf{K}\mathbf{A}_{DG}) \mathbf{V}_{DG_k} \tag{23}$$

$$\left[\begin{array}{c:cccc:cc} y_{1,1} & y_{1,2} & y_{1,3} & \cdots & y_{1,N} & y_{1,N+1} & y_{1,N+2} \\ \hdashline y_{2,1} & y_{2,2} & y_{2,3} & \cdots & y_{2,N} & y_{2,N+1} & y_{2,N+2} \\ \vdots & & & & & & \\ y_{N,1} & y_{N,2} & y_{N,3} & \cdots & y_{N,N} & y_{N,N+1} & y_{N,N+2} \\ \hdashline y_{N+1,1} & y_{N+1,2} & y_{N+1,3} & \cdots & y_{N+1,N} & y_{N+1,N+1} & y_{N+1,N+2} \\ y_{N+2,1} & y_{N+2,2} & y_{N+2,3} & \cdots & y_{N+2,N} & y_{N+2,N+1} & y_{N+2,N+2} \end{array}\right] \left[\begin{array}{c} V_S \\ \hdashline V_2 \\ \vdots \\ V_N \\ \hdashline V_{DG1} \\ V_{DG2} \end{array}\right] = \left[\begin{array}{c} I_S \\ \hdashline 0 \\ \vdots \\ 0 \\ \hdashline I_{DG1} \\ I_{DG2} \end{array}\right] \tag{15}$$

where $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $V_{DG_{k+1}}$ is the DG voltage at the next time step $(k+1)$, and V_{DG_k} is the DG voltage at current time step (k).

Equation (23) is in the form of $X_{k+1} = AX_k$ and therefore eigenvalue analysis of coefficient matrix A (where $A = I - KA_{DG}$) can predict the level of interaction and system instability contributed by the DGs. Eigenanalysis is useful for the analysis of small-signal stability of low frequency oscillations and for the design of corrective controls. The modes of oscillation can be clearly identified by eigenvalues of the system matrix (A-matrix) at which the damping and frequency of each mode change with different operating conditions. The examination of eigenvectors of individual modes helps to determine the characteristics of modes and assists in developing mitigating measures. Eigenanalysis for different DG locations and for different network loadings should be carried out to predict interaction and system instability caused by DGs. From (19), the indication of voltage interaction between two DGs in this example system can be observed and an interaction-index can be defined to predict the contribution of interaction by each DG. The diagonal elements of the matrix in (19) are the relative changes of self-voltages of DGs (changes at the DG connection points) located at particular location and off-diagonal elements in each row are the relative changes of voltages at other locations contributed by DG located in the position that produces relative change of self-voltage in that row. Therefore, the ratio of off-diagonal and diagonal elements in the column can be defined as an interaction-index that will indicate the interaction of DGs. For this example system, interaction-index for DG1 interacting with DG2 and interaction-index for DG2 interacting with DG1 are obtained as

$$Ind_{DG1}^{\text{Interact}} = \frac{V_{DG2_{N-2-1}}^{\text{Relative}}}{V_{DG1_{N-2-1}}^{\text{Relative}}}$$
$$Ind_{DG2}^{\text{Interact}} = \frac{V_{DG1_{N-6-1}}^{\text{Relative}}}{V_{DG2_{N-6-1}}^{\text{Relative}}}. \tag{24}$$

The interaction-index for DG$_i$ at a location L interacting with DG$_j$ at other location can be generalized as

$$Ind_{DGi|j}^{\text{Interact}} = \frac{V_{DGj_{L-1}}^{\text{Relative}}}{V_{DGi_{L-1}}^{\text{Relative}}}. \tag{25}$$

Interaction-index for a DG$_i$ at a location L interacting with $\mathrm{DG}_j, \mathrm{DG}_{j+1}, \mathrm{DG}_{j+2}, \ldots, \mathrm{DG}_{j+n}$ among multiple DGs can be generalized as shown in (26) at the bottom of the page.

The individual element of the matrix in (26) will indicate the degree of interaction for a DG with other DGs in different locations. It is noted that for a stable system, the value of interaction-index is very much less than 1.0 and close to zero. However, if the value of index approaches 1.0, this would indicate that the system is close to instability. There is a critical limit for every network, which depends on system parameters and loading. The network instability will occur if the network is loaded beyond this critical limit.

VII. Results and Discussions

A 120-km SWER network has been modeled based on the SWER systems in Western Queensland, Australia. The line parameters and line structure used in the model are similar to those in practice; however the uniform distributed load model is used for simplicity. The network has been formed with a single feeder of line impedance $Z_l = 1.828 + j0.876\ \mathrm{ohms/km}$. Source voltage is assumed $V_s = 19.1 + j0$ kV and source Thevenin impedance is $Z_s = 70.53 + j57.73$ ohms. $N = 20$ load buses have been considered in the SWER feeder. An isolating transformer with voltage regulation facility is connected at the beginning of SWER system. A voltage regulator is connected in the system to support network voltage. The high source impedance of the SWER system requires the regulation to be closer to the source [3]. A DG with synchronous generator will be installed on the SWER backbone to investigate the network dynamics and voltage improvement. The maximum voltage drop allowed in distribution systems is 6% as practiced in Australia [3].

A. Estimation of Voltage Sensitivity

Voltage sensitivity of the above SWER system is determined by using incremental power injection method. A 50-kVA DG is installed at position $N-2$ (where N is the bus number at far end). DG has been operated in DG-P and DG-Q modes with fixed tap and 352 kW load at 0.8 pf to inject maximum amount of power. The voltages at DG location and end node have been calculated and tabulated in Table I. The sensitivity for this test system is calculated from the increments of voltages due to DG-P and DG-Q and found as 1.8. Therefore, P and Q components are required in the ratio of 1.8:1 in steady state to improve the network voltage effectively. If DG-PQ operates at this ratio, it will provide a 15% improvement over a DG-P solution as seen in Table I.

B. Controlled Power Factor DG

For the above test system, the maximum amount of load with nominal fixed tap is found as 184 kW before the voltage limit of 0.94 p.u. is crossed. However, if automatic change of tap position is allowed, the maximum load is increased to 352 kW, for which case the power loss is obtained as 23.7 kW and voltage profile is shown in Fig. 4. Load power factor is assumed 0.8 lagging and kept constant in all cases. The delay time for tap operation is limited to 5 s to reduce the simulation time for this steady-state voltage study.

A 100-kVA DG is connected at position $N-2$ and operated to generate power in the P-Q ratio of maximum sensitivity for effective operation and voltage support. Fig. 5 shows the improvement of voltage profile for 352 kW load, for which case the power loss and lowest voltage in the feeder are obtained

$$Ind_{DGi|j,j+1,j+2,\ldots,j+n}^{\text{Interact}} = \begin{bmatrix} \frac{V_{DGj_{L-1}}^{\text{Relative}}}{V_{DGi_{L-1}}^{\text{Relative}}} & \frac{V_{DGj+1_{L-1}}^{\text{Relative}}}{V_{DGi_{L-1}}^{\text{Relative}}} & \frac{V_{DGj+2_{L-1}}^{\text{Relative}}}{V_{DGi_{L-1}}^{\text{Relative}}} & \cdots & \frac{V_{DGj+n_{L-1}}^{\text{Relative}}}{V_{DGi_{L-1}}^{\text{Relative}}} \end{bmatrix} \tag{26}$$

TABLE I
VOLTAGE SENSITIVITY

DG mode	P Gen.	Q Gen.	V_{DG}	V_{end}	ΔV at DG point	ΔV at end point	Sensitivity
No DG	0	0	0.9417	0.9400	-	-	-
DG-P	0.0500	0	0.9750	0.9733	0.0333	0.0333	1.8:
DG-Q	0	0.0500	0.9602	0.9586	0.0185	0.0186	1
DG-PQ	0.0437	0.0240	0.9800	0.9783	0.0383	0.0383	

Note: All are in p.u. and system base is 1-MVA and 19.1-kV

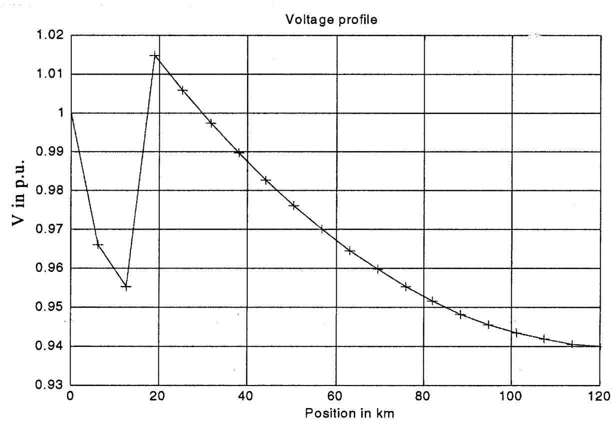

Fig. 4. Voltage profile with 352 kW load without DG.

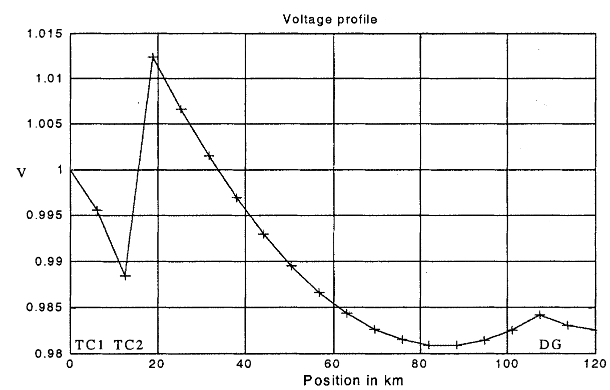

Fig. 5. Voltage profile with 352 kW load and 100-kVA DG (Power loss = 8.87 kW and lowest voltage = 0.9808 p.u.).

as 8.87 kW and 0.9808 p.u., respectively. Without violating the voltage limit, maximum load of 480 kW can be supported at the presence of 100-kVA DG. Fig. 6 shows how tap changers are controlling the voltage levels with 480 kW load for first 100 s of their operation. It is noted that the 480 kW load has been applied at the beginning of the 100-s period. The loading capacity and voltage profile of the network have been improved appreciably with DG. It is reported from the observation during simulation of this operation that no dynamic interaction between DG and tap changer is seen. This is because DG and tap changer are installed significantly distant from each other.

Two DGs, each with 50-kVA, have been considered for this paper to compare the results obtained for single DG with 100-kVA. DG1 is located at position $N-1$ and DG2 at the midpoint between regulator and DG1. Both are operated to

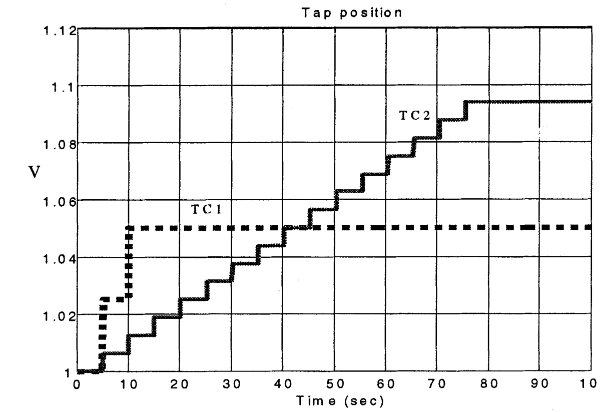

Fig. 6. Tap position to support maximum 480 kW load with 100-kVA DG.

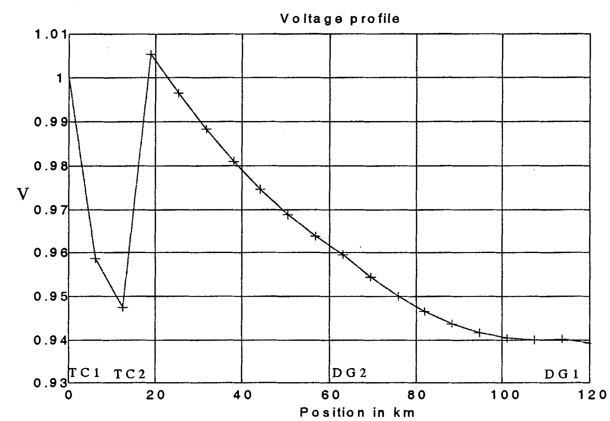

Fig. 7. Voltage profile with maximum 467 kW load and 2 × 50-kVA DGs (power loss = 21.5 kW).

support the network voltage. The maximum amount of 467 kW load can be supported by 2 × 50-kVA DG system and voltage profile for this load is shown in Fig. 7, whereas, 480 kW can be supported in the case of single DG with 100-kVA. The amount of maximum loading would be increased if the location of DG2 were moved toward DG1. It is observed that DG in single DG case provides better voltage support compared to the case of multiple DGs with same capacity of single DG. The performance could be better if the sizes of DGs in multiple DG are increased. No dynamic interactions of tap changer and DG or DG with another DG (DG-DG) are noticed during the operation of tap changer and DGs as they are constructed very far from each other and controlling voltages at different places of the network. For these results the full ratings of the DG are required with a high P component; thus the fuel penalty settings have been kept low to ensure the DG will run.

To investigate the interaction of the DG and the tap changer, DG2 position is moved to the bus adjacent to the regulator and DG1 position is fixed at $N-1$. The simulation is conducted for 352 kW load and voltage profile is shown in Fig. 8. No interaction between DG and tap changer is reported for this situation. In this case, DG is responding at every time step and tap changer is at every 5 s. The investigation is also conducted for the case of 1-s delay for tap changer, but no dynamic is noticed. Given the

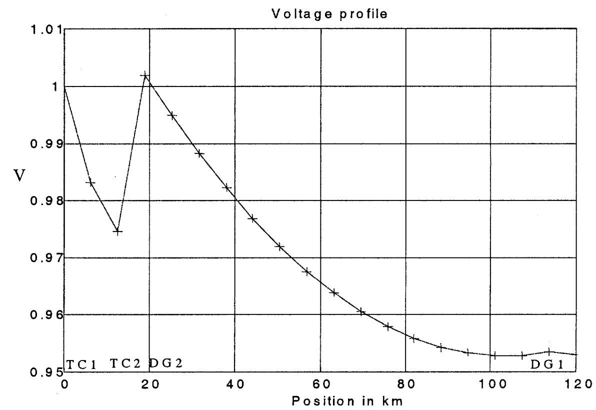

Fig. 8. One of the DGs and tap changer at close proximity with 352 kW load (power loss $= 13.6$ kW and lowest voltage $= 0.953$ p.u.).

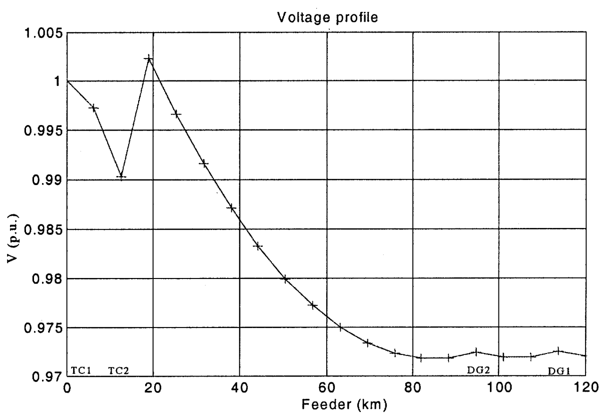

Fig. 9. DGs are at close proximity with 352 kW load (power loss $= 8.145$ kW and lowest voltage $= 0.9718$ p.u.).

lower voltage reference of the DG, the tap changer is expected to supply most of the voltage correction.

To determine the critical position of DG with a low level of interaction between two DGs, DG2 has been moved toward DG1 for the loading of 352 kW. The position of DG1 is kept fixed at $N-1$. It is observed from the simulation that DG2 at positions $N-2$ and $N-3$ has created instability and the whole system becomes unstable providing the large voltage oscillation. The best and critical position for DG2 is found as $N-4$ at which it exhibits optimum performance. The voltage profile for this position of DG2 is shown in Fig. 9. At this position of DG2, the real power loss and lowest voltage are found as 8.145 kW and 0.9718 p.u., respectively. For this case, both DGs are generating maximum currents. However, this position is very critical and the system may become unstable if loads are reduced or network parameters are changed. The integral part of the voltage control of the DGs creates problems if they are very close. This is particularly true if the reference voltage levels for the two DGs are not matched. In this case, one DG may reach its limit forcing the voltage up while the other limits forcing the voltage down. If two or more DGs are installed in the same location or bus, they may be locked with each other to avoid oscillatory interaction provided that they are identical and use common voltage controllers. When they are different in size, a set of current controllers may be used to precisely control the relative contributions. For large power station with multiple generators in parallel, there is usually little angle oscillation. Rotor dampers are sufficient to suppress any of these high frequency angle differences.

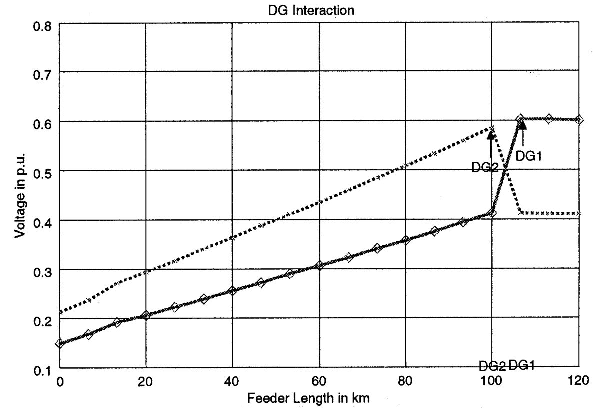

Fig. 10. Indication of voltage interaction with DG1 at position 18 and DG2 at 17 (total load of 330 kVA).

If the tap changer and DG are present during motor start, both of them may try to control the network voltage during a motor transient. However, the slow response of tap changer helps to avoid interaction between tap changer and DG. It is noted that the tap changer and DG should be installed far enough from each other to alleviate interaction between them. This is because, if they are installed in close proximity, they may not agree on the desired level for the local voltage and end with both controllers at opposite limits in the effort to control voltage.

C. Eigenanalysis and Voltage Interaction

Two DGs of 50 kVA each are installed on the SWER backbone to investigate the interaction of DGs. DG1 has been kept at position $N-2$ and the position of DG2 is being moved from buses 4 to $N-3$ (closest bus of DG1), one by one, to observe the interaction of DG-DG. The relative changes of voltages at DG connection points have been examined. Fig. 10 shows the relative changes of voltages at DG connection points for DG1 at position $N-2$ and DG2 at $N-3$, respectively. It is observed that when DG2 is coming in the proximity of DG1, the relative changes of voltages become closer. The interaction-indexes for DG1 and DG2 are calculated and graphically represented in Fig. 11. Interaction-indexes increase when separation distance between DG1 and DG2 is reduced. The position of DG1 is at $N-2$ where voltage dip is higher compared to the dip for the position of DG2. Therefore, DG1 has to generate more power to support the voltage. As the interaction-index depends on the relative values of connection point voltages of two DGs and the position at which the interaction is measured, the index values of two DGs are slightly different and for some cases DG2 has higher index value than DG1, even though they are identical. It is a fact that different sizes of DGs will have different index values. However, their position, of course, will play a role on interaction. DG interaction is also reflected on the changes of eigenvalues.

Fig. 12 shows the changes of eigenvalues for various separation distances between DG1 and DG2. As the analysis for

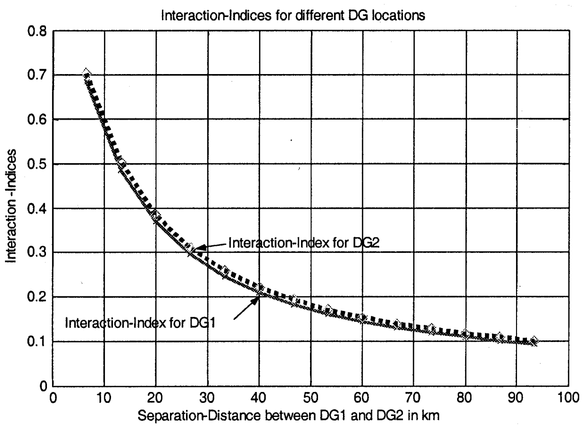

Fig. 11. Interaction indexes for DG1 and DG2 with different locations of DG2.

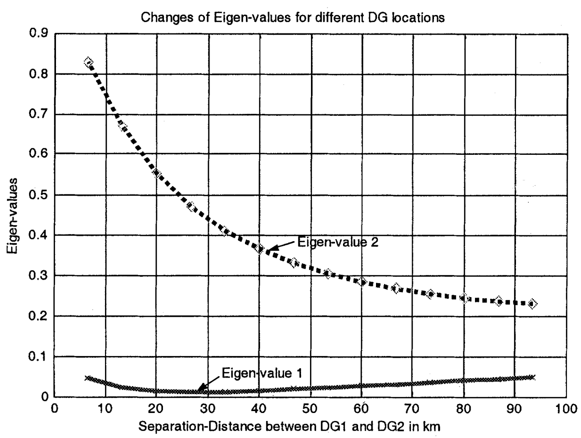

Fig. 12. Changes of eigenvalues for different DG locations.

voltage interaction of DGs has been carried out in discrete time domain, the largest eigenvalue will indicate the worst case interaction of DGs. For this case, it is eigenvalue 2 that indicates the interaction of DGs. From Fig. 12 it is seen that if the separation distance is small, the eigenvalue becomes large which may lead to voltage instability. The optimum separation distance depends on the individual system characteristics and system parameters and network loading. It is found that if both DGs become closer, voltage profile is improved to its maximum level. However, there is a risk of interaction between DGs for this situation. Therefore, the DG location should be determined by satisfying both requirements and using multiobjective optimization techniques. For this example SWER system, it is seen in the simulation that the minimum separation distance should be 18 km for which DGs give maximum improvement of voltage profile and low interaction. The interaction-indexes for both DGs at this point are closed to 0.4 and beyond this the system voltage profile becomes oscillatory and DGs introduce a high level of nonlinear interaction. For the value of interaction-index less than or equal to 0.4, the system with DG will operate smoothly and integral controllers of DGs will support voltage profile. For the higher value of interaction-index, a droop control system may be used in place of integral controller in DG system, which is capable of reducing the problem of interaction. Multiple DGs should be designed to contribute in the same proportion of their sizes for voltage correction in droop control system.

Changes of eigenvalues for different loading conditions with the position of DG1 kept at position $N-2$, and DG2 at positions $N-6$ and $N-4$, respectively, have been examined. For this study system, loading has been gradually increased from zero to 700 kVA. From the simulation it is observed that changes of eigenvalues due to system loading are very low for this example system and interaction-index remains low. Therefore, it can be concluded that while there is some influence on interaction eigenvalues from loading level, the loading effect has much lower influence than the proximity effect shown in Fig. 12.

It is found that if two DGs are in close proximity, the degree of interaction becomes stronger compared to the situation where they are separated with a considerable distance. The control action of controller can be decided from the results of off-line simulation and designed accordingly. The choice of control action fully depends on the size and parameters of network and also placement of DGs in the network which varies from network to network. The proposed interaction-index depends on network parameters and therefore different systems will have different index values. This index value represents an indication and degree of voltage interaction only and is not designed to determine the instability. Further research for extensive simulations on various networks with multiple DGs is required to establish the index value that is globally indicative of instability.

VIII. Conclusions

A system with distributed generation has greater load carrying capacity and can support peak-shaving. A network with DG can correct for poor voltage profile, especially needed during peak time of the day. This paper has assessed the operation and control of DG, and the dynamics of regulator-DG and DG-DG. Voltage sensitivity in distribution system has been performed to determine the operating point of DG. A control strategy has been developed for control of DG with real or reactive injection or both to support voltage. Dynamic interactions of DG-tap and DG-DG have been observed by installing DG at various places in the network.

In this study of an example SWER system, even though no interaction of DG and tap is found even when they are closely installed, it is suggested that DG should be placed far from a tap changer to have better voltage improvement. In this case of small DG, the voltage controller reference of DG2 is set such that DG does not tend to operate, as the regulator boosts the voltage at its connection point and makes the voltage level higher than the DG control voltage at DG connection. The interaction is observed for the case of DG-DG where they are placed nearby. Therefore, they should be placed far from each other. If they are required to be placed in close proximity, the DG controllers should be designed with droop controllers to avoid any interaction among them. While investing the interaction between DG and tap changer for their responses with different time delays, it is found that a fast DG and slow tap changer will be the safest solution at any conditions of the network. The DG also shows benefit in that addition of 100 kW of generation reduces the line loss by 15.7 kW and improves the lowest voltage in voltage profile by 4.1% for this test SWER system with 352-kW, 0.8-pf (lagging) load. Single DG and multiple DG cases also have been investigated and single DG gives a better solution for most of the cases. However, multiple DG solution provides benefits for long SWER lines with distributed load and motor starts as well as for reliability purposes. It is observed that DG operation with Q priority is most economical, as it requires generation of less energy and reduces the fuel requirement to meet the same level of voltage specification. For low levels of voltage correction, it has been found beneficial for the DG to operate with minimum real power injection and vary reactive injection from minimum to maximum. At higher levels of voltage correction, it is best to operate the DG at full rating with real and reactive injection. The DG controller needs to increase real injection and decrease reactive injection slowly and will settle at the point of maximum voltage sensitivity.

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M. A. Kashem (A'00–M'01) was born in Bangladesh in 1968. He received the Ph.D. degree from Multimedia University, Malaysia, in 2001.

He has four years' industrial experience and four years in teaching. Currently, he is a Lecturer at the School of Engineering, University of Tasmania, Australia. His special fields of interests include distributed generation, renewable energy, distribution system automation, power system planning, and artificial intelligence. He has published more than 30 technical papers in these areas.

Gerard Ledwich (M'73–SM'92) received the Ph.D. in electrical engineering from the University of Newcastle, Australia, in 1976.

He has been Chair Professor in Electrical Asset Management at Queensland University of Technology, Australia, since 1998. He was Head of electrical engineering at the University of Newcastle from 1997 to 1998. Previously he was associated with the University of Queensland from 1976 to 1994. His interests are in the areas of power systems, power electronics, and controls.

Prof. Ledwich is a Fellow of I.E.Aust.